January 1995

Mathematics 30

Grade 12 Diploma Examination

Alberta EDUCATION

January 1995

Mathematics 30

Grade 12 Diploma Examination

Description

Time: 2.5 h.
You may take an additional 0.5 h to complete the exam.

Total possible marks: 70

This is a **closed-book** examination consisting of **three** parts:

Part A
has 40 multiple-choice questions each with a value of one mark.

Part B
has 9 numerical-response questions each with a value of one mark.

Part C
has 4 written-response questions for a total of 21 marks.

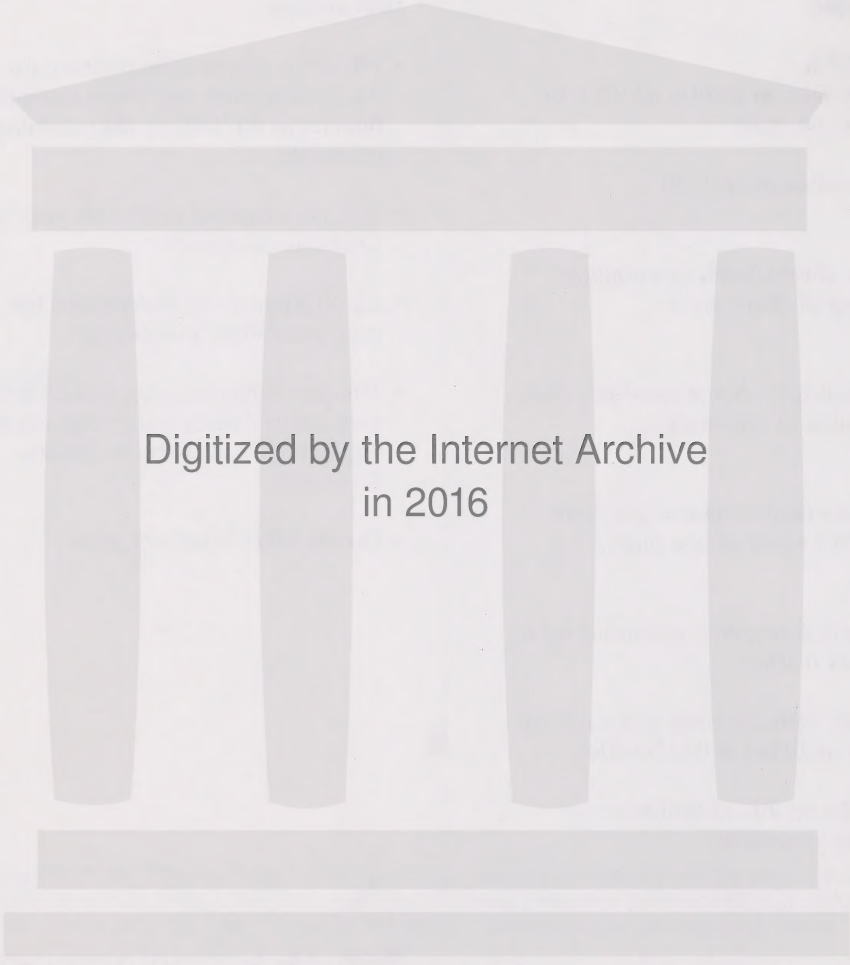
A tear-out formula sheet and a z-score page are included in this booklet.

All graphs on this examination are computer-generated.

Instructions

- Fill in the information required on the answer sheet and the examination booklet as directed by the presiding examiner.
- You are expected to provide your own scientific calculator.
- Carefully read the instructions for each part before proceeding.
- The presiding examiner will collect your answer sheet and examination booklet and send them to Alberta Education.
- Do not fold the answer sheet.

Note: *The perforated pages at the back of this booklet may be torn out and used for your rough work.*
No marks *will be given for work done on the tear-out pages.*



Part A: Multiple Choice

40 Questions

Instructions

- Consider all numbers used in the questions to be **exact real** numbers and not the result of a measurement.
- Read each question carefully and decide which of the choices **best** completes the statement or answers the question.
- Locate that question number on the separate answer sheet provided and fill in the circle that corresponds to your choice.

Example

This diploma examination is for the subject of

A. biology
B. physics
C. chemistry
D. mathematics

Answer Sheet

Ⓐ Ⓑ Ⓒ ●

- **Use an HB pencil only.**
- If you wish to change an answer, erase **all** traces of your first answer.

Note: *The perforated pages at the back of this booklet may be torn out and used for your rough work.* ***No marks*** *will be given for work done on the tear-out pages.*

Do not turn the page to start the examination until told to do so by the presiding examiner.

1. $P(x)$ is a third-degree polynomial with zeros of 2, –2, and –5. $P(x)$ could be

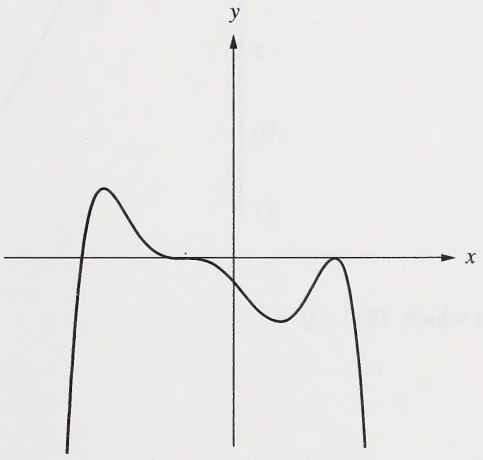
A. $x^3 - 5x^2 + 4x - 20$

B. $x^3 - 5x^2 - 4x - 20$

C. $x^3 + 5x^2 - 4x - 20$

D. $x^3 + 5x^2 + 4x + 20$

2. The graph of a polynomial function is shown below.



If all the x-intercepts are shown, then the minimum degree of this polynomial function is

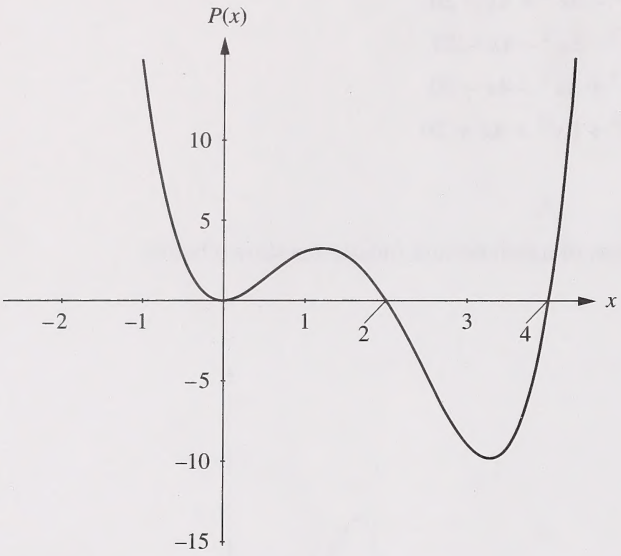
A. 1

B. 3

C. 4

D. 6

3. The graph of the polynomial function $P(x) = x^2(x-2)(x-4)$ is shown below.



The domain in which $P(x) < 0$ is

A. $x < 0$

B. $x > 0$

C. $2 < x < 4$

D. $x < 2$ or $x > 4$

4. When $P(x) = x^3 + kx^2 - x - 30$ is divided by $x + 2$, the remainder is -12. If $P(x)$ is divided by $x - 3$, then the remainder is

A. 48

B. 6

C. –27

D. –81

5. Which of the following statements about an integral polynomial function is always true?

A. The terms of the polynomial have only integral coefficients.

B. All the x-intercepts of the graph of the function are integers.

C. The only term in the polynomial that must be an integer is the constant term.

D. The number of x-intercepts of the graph of the function is equal to the degree of the polynomial.

6. When $(x + 2)(x^2 - 2x + 4)$ is divided by $(x + 2)$, where $x \neq 2$, the remainder is

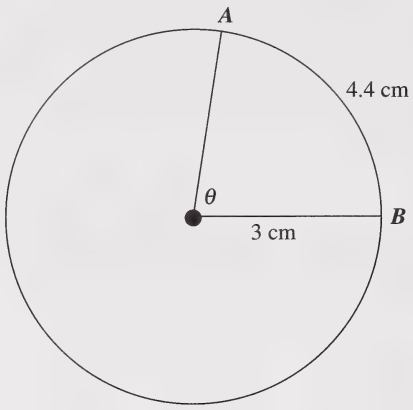
A. 0

B. -2

C. $x + 2$

D. $x^2 - 2x + 4$

7. In the diagram shown below, the length of arc AB is 4.4 cm. The radius of the circle is 3 cm.



The measure of θ correct to the nearest degree is

A. 132°
B. 96°
C. 84°
D. 39°

8. If $f(\theta) = \sin\theta$, then the range of $f(\theta) - 1$ is

A. $-1 \leq f(\theta) \leq 1$
B. $-2 \leq f(\theta) \leq 0$
C. $\theta \geq -1$
D. $\theta \in R$

9. The expression $\dfrac{1}{1-\sec^2\theta}$ is equivalent to

A. $-\tan^2\theta$

B. $-\cot^2\theta$

C. $-\csc^2\theta$

D. $-\sin^2\theta$

10. If $\sin\theta=\dfrac{5}{6}$, then $\cos\left(\dfrac{\pi}{2}+\theta\right)$ is

A. $-\dfrac{5}{6}$

B. $-\dfrac{\sqrt{11}}{6}$

C. $\dfrac{\sqrt{11}}{6}$

D. $\dfrac{5}{6}$

11. An angle with radian measure 1.31 has a degree measure of

A. 235.8°

B. 150.1°

C. 75.1°

D. 4.1°

12. The expression $\dfrac{\sin\theta\cot\theta+\cos\theta}{\sin\theta}$ is equivalent to

A. $\cot\theta+\cos\theta$

B. $2\cos\theta$

C. $2\cot\theta$

D. $\cot 2\theta$

13. If $\sin\theta = \frac{3}{4}$ or $\sin\theta = \frac{1}{2}$, then a quadratic equation that could have the same solutions is

A. $\sin^2\theta - 4\sin\theta + 3 = 0$

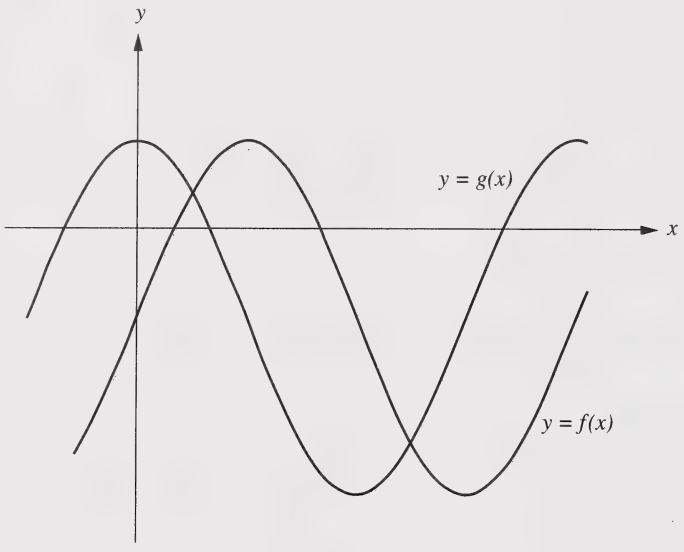
B. $\sin^2\theta + 4\sin\theta + 3 = 0$

C. $8\sin^2\theta + 10\sin\theta + 3 = 0$

D. $8\sin^2\theta - 10\sin\theta + 3 = 0$

Use the following information to answer the next question.

The graphs of two trigonometric functions are shown below.



14. The solutions to $f(x) \times g(x) = 0$ would be the

A. y-intercepts of $y = f(x)$ or $y = g(x)$

B. x-intercepts of $y = f(x)$ or $y = g(x)$

C. maximum values of $y = f(x)$ and $y = g(x)$

D. points of intersection of $y = f(x)$ and $y = g(x)$

15. Written in logarithmic form, the equation $49^{\frac{1}{2}} = 7$ is

A. $\log_{\frac{1}{2}}(7) = 49$

B. $\log_{49}\left(\frac{1}{2}\right) = 7$

C. $\log_{7}(49) = \frac{1}{2}$

D. $\log_{49}(7) = \frac{1}{2}$

16. In order to determine its value, $\log_3 30$ can be written as

A. $\frac{\log_{10}30}{\log_{10}3}$

B. $\frac{\log_{10}3}{\log_{10}30}$

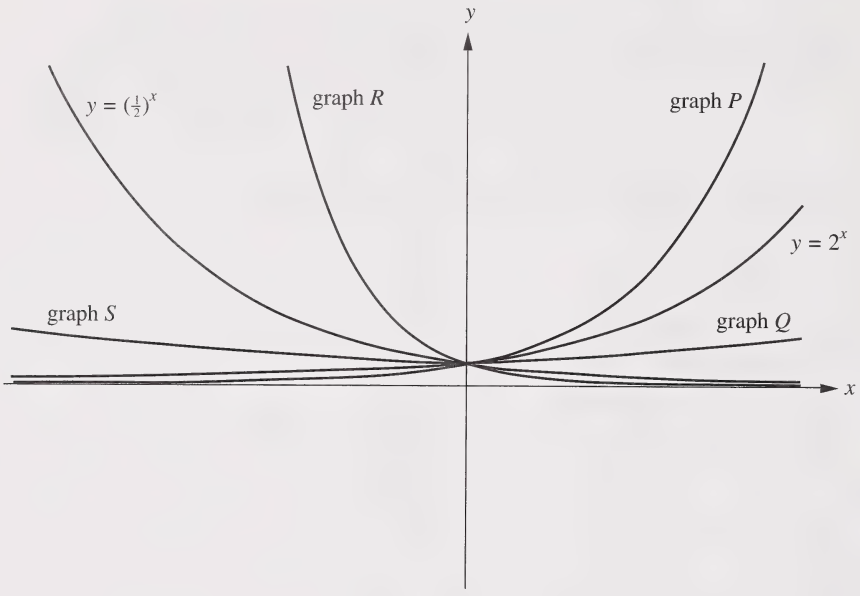
C. $\log_{10}30 - \log_{10}3$

D. $\log_{10}10$

Use the following information to answer the next question.

One commonly known irrational number is π. Another irrational number, called the Euler number, is represented by the letter e. The value of e is approximately 2.7.

17. The graphs of six exponential functions are shown below.



If one of the graphs is the graph of $y = e^x$, then it can only be

A. graph P
B. graph Q
C. graph R
D. graph S

18. If $\log_{10}(x) + \log_{10}(x) + \log_{10}(x) = 27$, then x is

A. 3
B. 9
C. 10^3
D. 10^9

Use the following information to answer the next question.

A scientist observing a bacteria culture arrived at the following conclusions:

I. the bacterial population at time $t = 0$ was 100
II. the bacterial population increases exponentially
III. the bacterial population doubles every hour

19. The equation that best represents the bacterial population growth $P(t)$ after t hours is

A. $P(t) = 200^t$
B. $P(t) = 200^{2t}$
C. $P(t) = 100(2^t)$
D. $P(t) = 2(100)^t$

---

20. If $a^{x+3} = b^{-1}$, then the value of x is

A. $\log_b(a) - 3$

B. $\log_a\left(\frac{1}{b}\right) - \log_a(3)$

C. $\frac{-[3b \log_{10}(a) - 1]}{b \log_{10}(a)}$

D. $\frac{-[\log_{10}(b) + 3 \log_{10}(a)]}{\log_{10}(a)}$

Use the following information to answer the next question.

Over 400 years ago, Kepler determined that the planets move in an elliptical path where the sun is at one focus. The length of a planet's year T is related to the semi-major axis of its orbit D by

$$3 \log_{10}(D) = 2 \log_{10}(T) + 1.4, \text{ where}$$

D is the distance in millions of kilometres

T is the time in days

21. If the semi-major axis of Jupiter's orbit is 7.78×10^8 km, then the length of its year is approximately

A. 4330 d

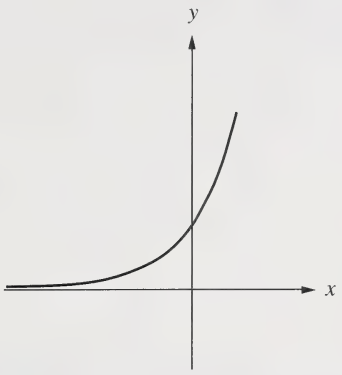
B. 4060 d

C. 3670 d

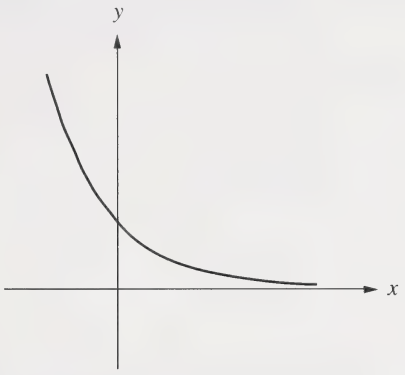
D. 3240 d

22. The graph of $y = \log_{\frac{1}{3}} x$ is best represented by

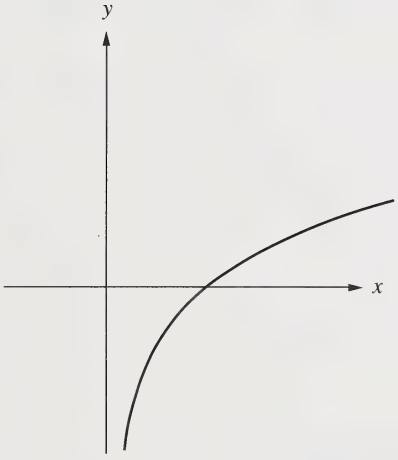
A.



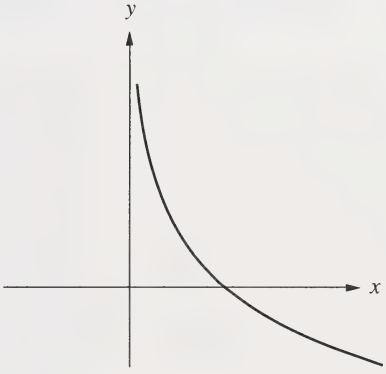
B.



C.



D.



23. Another way of expressing the sum of $1 + 3 + 5 + 7 + 9$ is

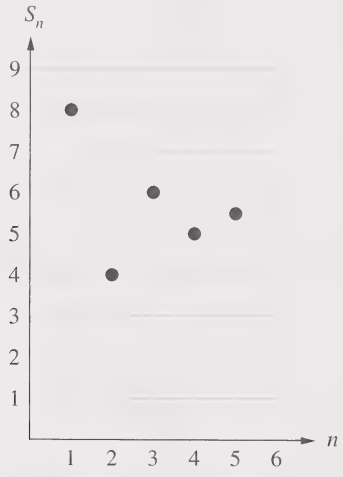
A. $\sum_{k=1}^{5}(2k-1)$

B. $\sum_{k=1}^{9}(2k-1)$

C. $\sum_{k=3}^{5}(2k-5)$

D. $\sum_{k=3}^{9}(2k-5)$

24. The graph below plots the sum of n terms of a geometric sequence as a function of n, $n \in N$. Each point on the graph is in the form (n, S_n) where $n \in N$ and S_n is the sum of n terms of the sequence. For example, the point (3, 6) represents $S_3 = 6$.



The first five terms in the geometric sequence are

A. 1, 2, 4, 8, 16

B. 1, –2, 4, –8, 16

C. 8, –4, 2, –1, $\frac{1}{2}$

D. 8, 4, 2, 1, $\frac{1}{2}$

25. Fifteen members of a relay team compete in a race. Each member runs to a marker and then back to the starting line. The first marker is 10 m from the starting line, and each successive marker is 5 m farther away. If the first member runs to the first marker and back, the second member runs to the second marker and back, and so on, what fraction of the total possible distance is completed by the first nine members?

A. $\frac{2}{5}$

B. $\frac{3}{5}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

26. The common difference of the sequence defined by $t_n = \frac{1}{2}(5-n),\ n \in N$ is

A. $-\frac{5}{2}$

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. $\frac{5}{2}$

27. The degenerate case of the hyperbola is a locus consisting of

A. one line

B. one point

C. two parallel lines

D. two intersecting lines

28. If $P(x, y)$ is any point on a parabola with a focus $(3, 0)$ and directrix $x = -3$, then (x, y) satisfies

A. $\sqrt{(x+3)^2 + y^2} = \sqrt{(x-3)^2}$

B. $\sqrt{(x-3)^2 + y^2} = \sqrt{(x+3)^2}$

C. $\sqrt{x^2 + (y+3)^2} = \sqrt{(y-3)^2}$

D. $\sqrt{x^2 + (y-3)^2} = \sqrt{(y+3)^2}$

29. A circular conical surface is intersected by a plane. The plane is perpendicular to the axis of the cone but does **not** pass through the vertex. The conic section formed by the plane cutting the conical surface is

A. a circle

B. an ellipse

C. a parabola

D. a hyperbola

30. If F_1 and F_2 are fixed points and P is any point on a locus such that $PF_1 + PF_2$ is a constant, then the curve is

A. a hyperbola

B. a parabola

C. an ellipse

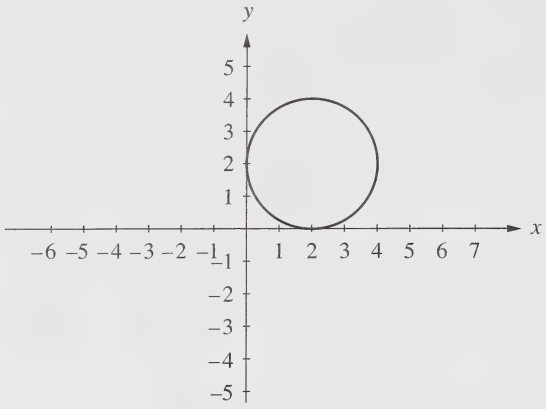
D. a circle

Use the following information to answer the next question.

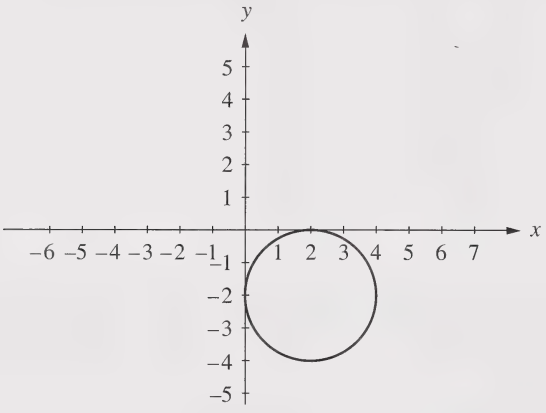
31. Which of the points, R, S, T, or U, will lie on a hyperbola with the given focus and directrix?

A. R
B. S
C. T
D. U

32. A computer graphing utility is used to obtain the graph of the circle shown below.



The following graph is obtained when the circle is reflected across the x-axis.



The original equation entered into the computer was in the form $x^2 + y^2 + Dx + Ey + F = 0$. When the circle is reflected across the x-axis, there is a change to the value of

A. E and F only

B. D, E, and F

C. D only

D. E only

33. A necessary condition for the graph of the quadratic relation $Ax^2 + Cy^2 + Dx + Ey + F = 0$ to be a parabola is

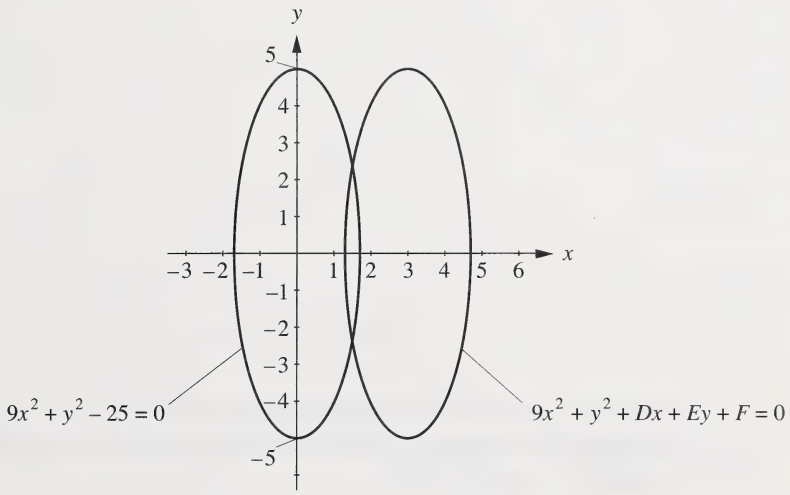
A. $A = C$

B. $AC > 0,\ A \neq C$

C. $AC < 0,\ A \neq C$

D. $AC = 0,\ A \neq C$

34. The graph of $9x^2 + y^2 - 25 = 0$ and the graph of $9x^2 + y^2 + Dx + Ey + F = 0$ are shown below.



If the graph of $9x^2 + y^2 + Dx + Ey + F = 0$ is the graph of $9x^2 + y^2 - 25 = 0$ moved 3 units to the right, then the conditions satisfied by the values of D, E, and F are

A. $D > 0,\ E < 0,\ F \neq -25$

B. $D < 0,\ E = 0,\ F \neq -25$

C. $D < 0,\ E < 0,\ F = 25$

D. $D > 0,\ E = 0,\ F = 25$

35. An examination consists of 2 parts. Part A has 3 questions and Part B has 6 questions. A student completing the examination must do all the questions in Part A and only 4 questions in Part B. How many ways can the student select the 7 questions?

A. 6

B. 15

C. 24

D. 360

36. Bill, Sandra, and five other students stand in a row. The probability that Bill and Sandra do **not** stand next to one another is

A. $\frac{6}{7}$

B. $\frac{5}{7}$

C. $\frac{2}{7}$

D. $\frac{1}{7}$

37. A committee of 4 will be selected from a group of 5 girls and 2 boys. Girls and boys must be represented on the committee. If this restriction is in place, then how many different committees are possible?

A. 12

B. 21

C. 30

D. 70

38. A manufacturing company finds that the masses of its chocolate bars are normally distributed with a mean of 80 g and a standard deviation of 10 g. The number of chocolate bars that have a mass greater than 70 g is equal to the number of chocolate bars with a mass

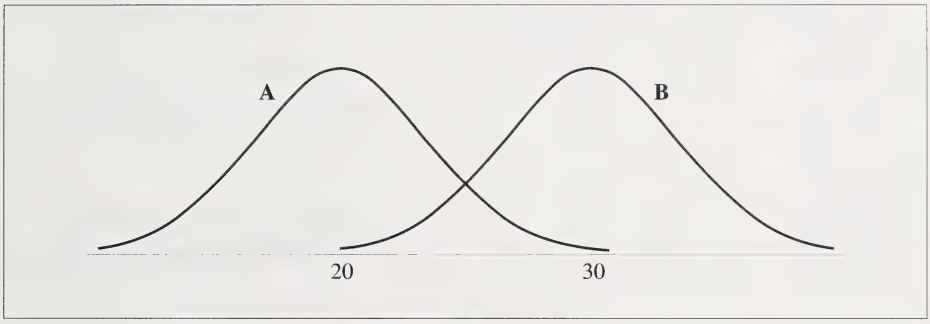
A. less than 70 g

B. less than 90 g

C. greater than 80 g

D. greater than 90 g

Use the following information to answer the next question.



39. Based on this diagram, which statement is **true**?

A. The mode of distribution *A* is less than the mode of distribution *B*.

B. The mode of distribution *A* is the same as the mode of distribution *B*.

C. The median of distribution *A* is greater than the median of distribution *B*.

D. The median of distribution *A* is the same as the median of distribution *B*.

40. The heights of male students in a school were found to be normally distributed. If one of these students is selected at random, then the probability that his height differs from the mean height by at least 1.6 standard deviations is

A. 0.0548

B. 0.0636

C. 0.1096

D. 0.1272

You have now completed Part A. Proceed directly to Part B.

Part B: Numerical Response

9 Questions

Instructions

- Consider all numbers used in the questions to be **exact positive real** numbers and not the result of a measurement.
- Read each question carefully.
- Record your answer on the answer sheet provided by writing it in the boxes and then filling in the corresponding circles.
- **Enter the first digit of your answer in the left-hand box and leave any unused boxes blank.**
- Use an HB pencil only.
- If you wish to change an answer, erase **all** traces of your first answer.

Sample Questions and Solutions

Correct to the nearest tenth of a radian, 40° is equal to _____ rad.

$40^\circ = 0.6981317008\ldots$ rad

Record 0.7 on the answer sheet →

For the arithmetic series $-8 + (-5) + (-2) + \ldots + (85)$, the number of terms is _____.

$85 = -8 + (n - 1)(3)$

$93 = 3n - 3$

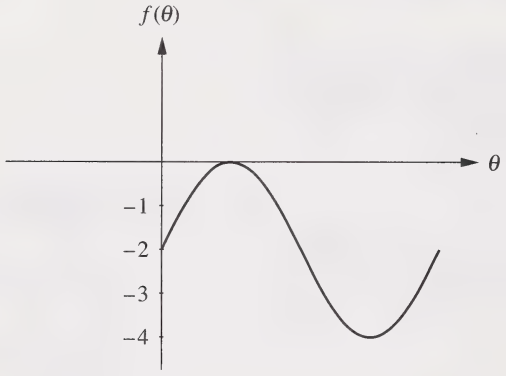
$n = 32$

Record 32 on the answer sheet →

Start Part B immediately.

1. An ice cream parlour offers a "Super Deal" for an ice cream dessert that consists of a cone, one scoop of ice cream, and a topping. There are 56 flavours of ice cream, 3 different toppings, and 3 types of cones. The number of different "Super Deal" ice cream desserts that could be ordered is _____ .
(Record your answer on the answer sheet.)

2. The partial graph of a translated sine function is shown below.



The amplitude of this sine function appears to be _____ .
(Record your answer on the answer sheet.)

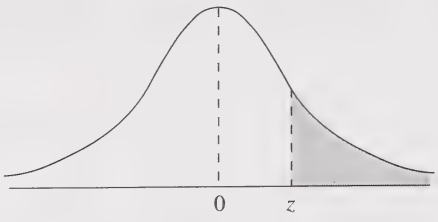
3. The number of terms in the expansion of the binomial $(2x + 7)^4$ is _____ .
(Record your answer on the answer sheet.)

4. If $5^{x+7} = 5^{4(x-3)}$, then the value of x correct to the nearest hundredth is _____ .
(Record your answer on the answer sheet.)

5. If $\log_4(2x + 3) - \log_4 x = 2$, then the value of x correct to the nearest hundredth is _____ .
(Record your answer on the answer sheet.)

6. On a provincial achievement test, the results were normally distributed with a mean of 61.8 and a standard deviation of 8.0. The test was written by 11 500 students. The number of students with a score greater than 83 was _____ .
(Record your answer on the answer sheet.)

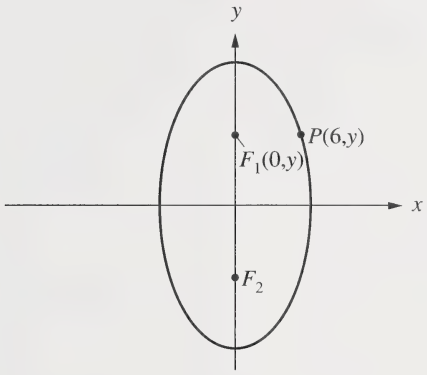
7. The shaded area in the standard normal curve shown below is 0.1151.



The corresponding z-score correct to the nearest tenth is _____ .
(Record your answer on the answer sheet.)

8. A point on the graph of an ellipse is $P(-6, 4)$. The focus of this ellipse is at $F(-3, 0)$ and its corresponding directrix is $x = 1$. Correct to the nearest hundredth, the value of the eccentricity of this ellipse is _____ .
(Record your answer on the answer sheet.)

9. The centre of the ellipse shown below is at the origin. One of its foci F_1 has coordinates $(0, y)$. The constant sum for this ellipse is 16.



If the ellipse passes through $P(6, y)$, then the value of y correct to the nearest tenth is _____ .
(Record your answer on the answer sheet.)

You have now completed Part B. Proceed directly to Part C.

Part C: Written Response

4 Questions

Instructions

- Consider all numbers used in the question to be **exact real** numbers and not the result of a measurement.
- Read each question carefully.
- Write your answers in the examination booklet as neatly as possible.
- For full marks, your answers **must show all** pertinent explanations, calculations, and formulas.
- Your answers **should be** presented in a well-organized manner using complete sentences for a written response and correct units for a numerical response.

Note: *The perforated pages at the back of this booklet may be torn out and used for your rough work.* ***No marks*** *will be given for work done on the tear-out pages.*

Start Part C immediately.

For Department Use Only

(4 marks)

1. $2\sin^2\theta - 5\sin\theta + 2 = (2\sin\theta - 1)(\sin\theta - 2)$ for all θ.

a. Determine when $2\sin^2\theta - 5\sin\theta + 2 = 0, \quad 0 \le \theta < 2\pi$.

b. Describe how the solution of

$$2\sin^2(2\theta) - 5\sin(2\theta) + 2 = 0, \quad 0 \le \theta < 2\pi$$

relates to the solution of

$$2\sin^2\theta - 5\sin\theta + 2 = 0, \quad 0 \le \theta < 2\pi.$$

For Department Use Only

(7 marks)

2. In the event that the school is closed for unforeseen reasons, the principal would like to use a "telephone fan-out system" to contact all staff members.

In this "telephone fan-out system," the principal telephones n people and then each one of these people telephones n more people. This pattern continues until all staff members are telephoned.

- Draw and label a diagram that models such a "telephone fan-out system."

- An equation can be determined to describe the total number of staff members phoned. From your model, determine an equation that connects the total number of people phoned with the value of the variable n. Introduce and define any other variables, as required, to complete your equation.

- What assumption is necessary to construct a diagram that models such a "telephone fan-out system."

- The principal asks you to design a "telephone fan-out system" so that all staff are telephoned in the least amount of time. What factors will you have to consider so that the system you design will take the least amount of time?

(Question 3 is on the next page.)

For Department Use Only

(5 marks)

3. Jennifer was given the following problem to solve.

Eight students volunteer to help out at the school dance. Two volunteers are required to check coats, two different volunteers are required to sell pop, and the remaining volunteers are required to clean the gym. How many different ways can the students be selected for the various jobs?

Jennifer considered two models that correctly represent this problem:

Model A	Model B
${}_8C_2 \times {}_6C_2 \times {}_4C_4$	$\frac{8!}{2!2!4!}$

Describe how each model represents the problem.

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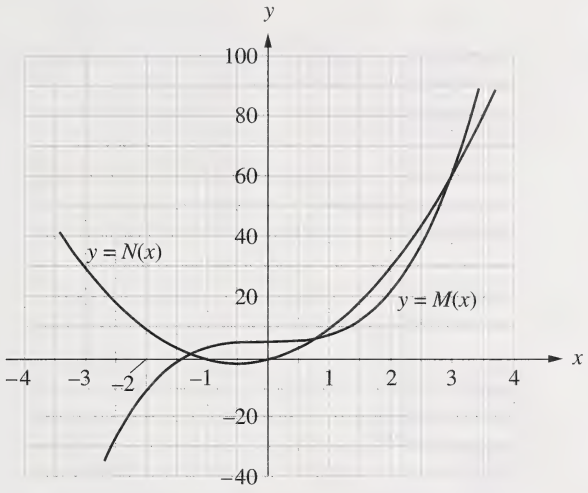
(5 marks)

4. All the points of intersection of the graphs of $y = M(x) = 2x^3 + 6$ and $y = N(x) = 5x^2 + 5x$ are shown below.



Find the **exact values** of x for which $M(x) = N(x)$.

You have now completed the examination.
If you have time, you may wish to check your answers.

Mathematics 30 Formula Sheet

The following information may be useful in writing this examination.

- The roots of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The distance between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Quadratic Relations

- $e = \dfrac{|\overline{PF}|}{|\overline{PD}|}$

Trigonometry

- arc length $a = r\theta$
- $\sin^2 A + \cos^2 A = 1$
- $1 + \tan^2 A = \sec^2 A$
- $1 + \cot^2 A = \csc^2 A$
- $\csc A = \dfrac{1}{\sin A}$
- $\sec A = \dfrac{1}{\cos A}$
- $\cot A = \dfrac{\cos A}{\sin A}$
- $\sin(A + B) = \sin A \cos B + \cos A \sin B$
- $\sin(A - B) = \sin A \cos B - \cos A \sin B$
- $\cos(A + B) = \cos A \cos B - \sin A \sin B$
- $\cos(A - B) = \cos A \cos B + \sin A \sin B$

Permutations and Combinations

- ${}_nP_r = \dfrac{n!}{(n-r)!}$
- ${}_nC_r = \dfrac{n!}{r!(n-r)!}$
- In the expansion of $(x + y)^n$, the general term is $t_{k+1} = {}_nC_k x^{n-k} y^k$

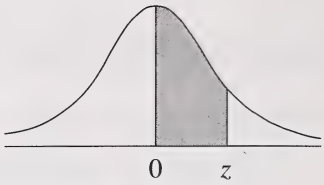
Sequences and Series

- $t_n = a + (n-1)d$
- $S_n = \dfrac{n[2a + (n-1)d]}{2}$
- $S_n = n\left(\dfrac{a + t_n}{2}\right)$
- $t_n = ar^{n-1}$
- $S_n = \dfrac{a(r^n - 1)}{r - 1}$, $r \neq 1$
- $S_n = \dfrac{rt_n - a}{r - 1}$, $r \neq 1$

Exponential and Logarithmic Functions

- $\log_a mn = \log_a m + \log_a n$
- $\log_a \dfrac{m}{n} = \log_a m - \log_a n$
- $\log_a m^n = n \log_a m$

$$z = \frac{x - \mu}{\sigma}$$



Areas under the Standard Normal Curve

z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

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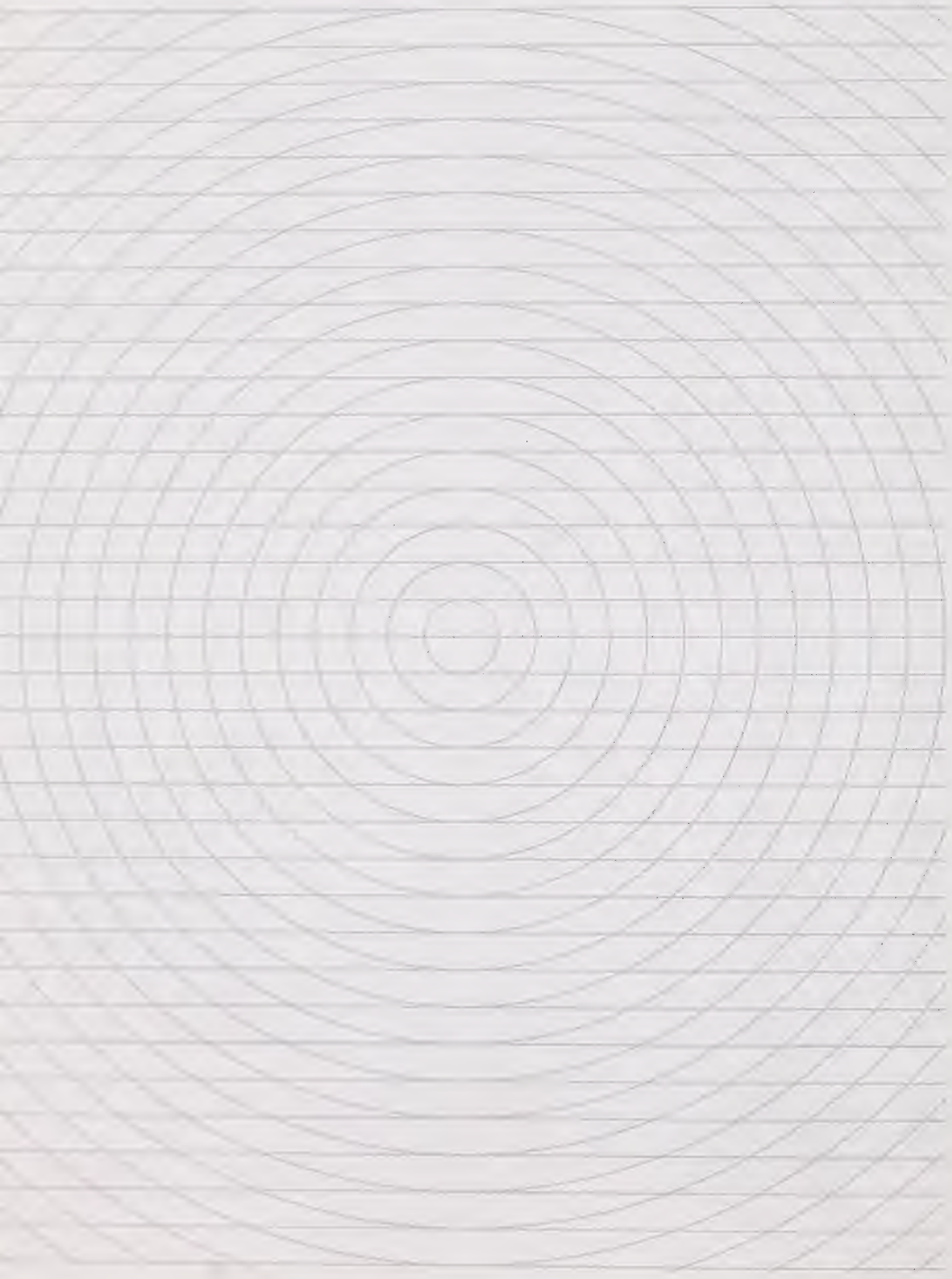

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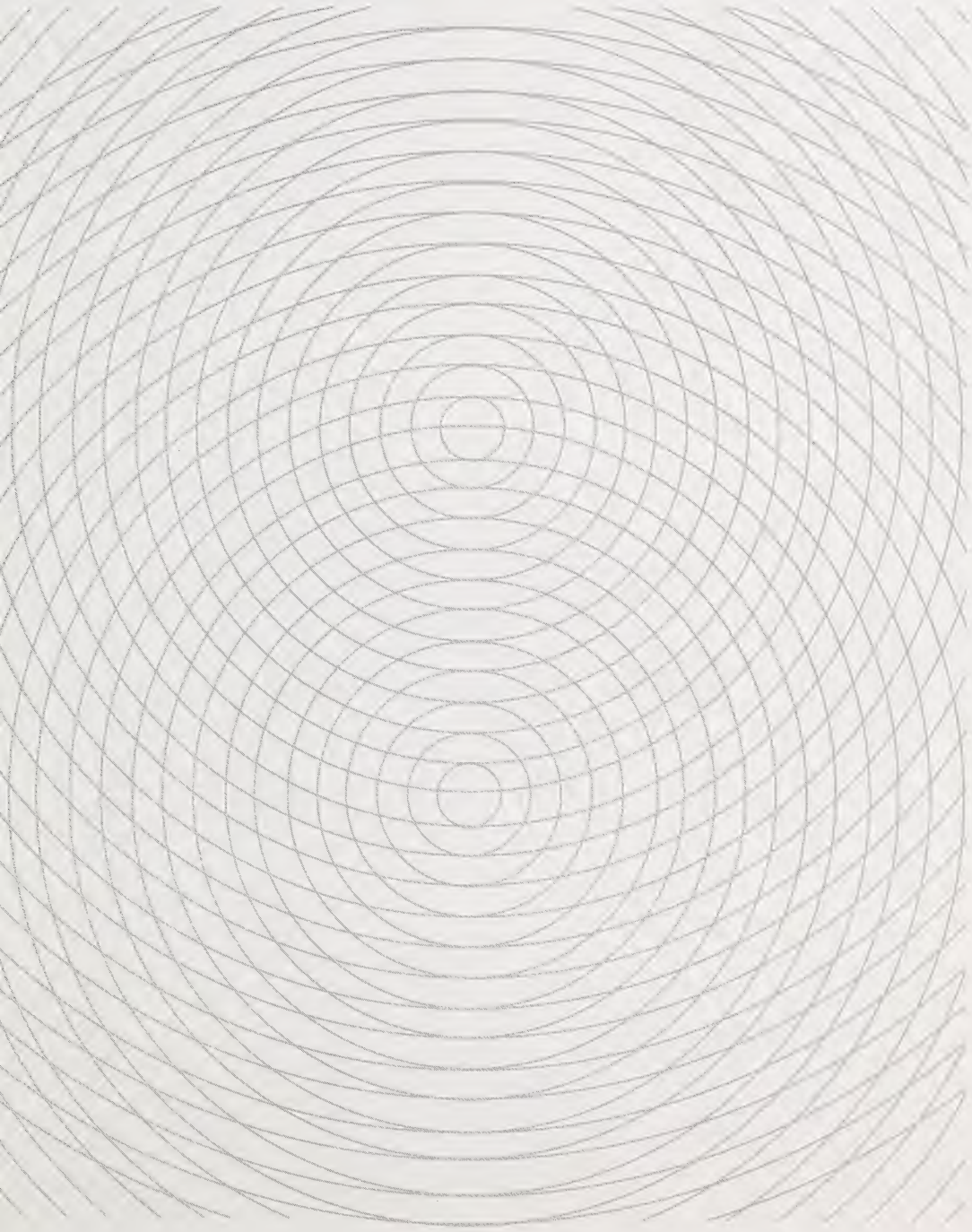

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Mathematics 30
January 1995

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Mathematics 30 Diploma Examination January, 1995

Multiple-Choice Key, Numerical-Response Key and Sample Answers to Written-Response Questions

MATHEMATICS 30 - 951

MULTIPLE-CHOICE KEY

1.	C	21.	A
2.	D	22.	D
3.	C	23.	A
4.	A	24.	C
5.	A	25.	A
6.	A	26.	B
7.	C	27.	D
8.	B	28	B
9.	B	29.	A
10.	A	30.	C
11.	C	31.	D
12.	C	32.	D
13.	D	33.	D
14.	B	34.	B
15.	D	35.	B
16.	A	36.	B
17.	A	37.	C
18.	D	38.	B
19.	C	39.	A
20.	D	40.	C

NUMERICAL-RESPONSE KEY

1.	504
2.	2
3.	5
4.	6.33
5.	0.21
6.	46
7.	1.2
8.	0.71
9.	4.0

SAMPLE ANSWERS TO THE WRITTEN-RESPONSE SECTION

Note: The responses that follow represent ONE approach to each of the problems. During the diploma examination marking session, provision is made for considering the various approaches students may have used.

(4 marks) 1. $2\sin^2\theta - 5\sin\theta + 2 = (2\sin\theta - 1)(\sin\theta - 2)$ for all θ.

a. Determine when $2\sin^2\theta - 5\sin\theta + 2 = 0,\ 0 \le \theta < 2\pi$.

Sample Answer Question 1a

$\sin\theta = \frac{1}{2}$ OR $\sin\theta = 2$

$\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ no solution since $-1 \le \sin\theta \le 1$

---

b. Describe how the solution of

$$2\sin^2(2\theta) - 5\sin(2\theta) + 2 = 0,\ 0 \le \theta < 2\pi$$

relates to the solution of

$$2\sin^2\theta - 5\sin\theta + 2 = 0,\ 0 \le \theta < 2\pi.$$

Sample Answer Question 1b

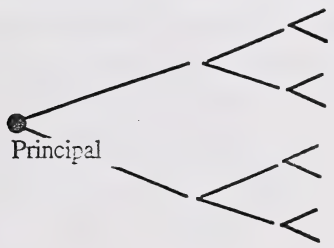
The period of the graph of $y = 2\sin^2 2\theta - 5\sin 2\theta + 2$ is π compared to the period of the graph of $y = 2\sin^2\theta - 5\sin\theta + 2$ which is 2π. The θ intercepts are the solutions to $0 = 2\sin^2 2\theta - 5\sin 2\theta + 2$ so there will be twice as many solutions. The values of the solution to $0 = 2\sin^2 2\theta - 5\sin 2\theta + 2$ will be $\frac{1}{2}$ of the values to the solution to $0 = 2\sin^2\theta - 5\sin\theta + 2$.

(7 marks) **2.** In the event that the school is closed for unforeseen reasons, the principal would like to use a "telephone fan-out system" to contact all staff members.

In this "telephone fan-out system," the principal telephones n people and then each one of these people telephones n more people. This pattern continues until all staff members are telephoned.

- Draw and label a diagram that models such a "telephone fan-out system."
- An equation can be determined to describe the total number of staff members phoned. From your model, determine an equation that connects the total number of people phoned with the value of the variable n. Introduce and define any other variables, as required, to complete your equation.
- What assumption is necessary to construct a diagram that models such a "telephone fan-out system."
- The principal asks you to design a "telephone fan-out system" so that all staff are telephoned in the least amount of time. What factors will you have to consider so that the system you design will take the least amount of time?

Sample Answer Question 2



- Let k = The number of people the principal phoned. (both the first term and ratio)

  Let n = The number of rounds. The first round would be when the principal has contacted n people and the second round is where these people have each contacted n people.

  S_n = total number of staff members phoned

  $S_n = \frac{k(k^n - 1)}{k - 1}$

- An assumption necessary to construct a diagram is that you know the number of staff members that need to be phoned.
- We assume that everyone can be contacted in 2 minutes or less. If someone was not at home or a phone call took more than 2 minutes our model would not accurately reflect the situation.

(5 marks) **3.** Jennifer was given the following problem to solve.

Eight students volunteer to help out at the school dance. Two volunteers are required to check coats, two different volunteers are required to sell pop, and the remaining volunteers are required to clean the gym. How many different ways can the students be selected for the various jobs?

Jennifer considered two models that correctly represent this problem:

Model A	Model B
${}_8C_2 \times {}_6C_2 \times {}_4C_4$	$\frac{8!}{2!2!4!}$

Describe how each model represents the problem.

Sample Answer Question 3

Model A

Jennifer used the Fundamental Counting Principle.

${}_8C_2$ represents choosing 2 people from the eight to check coats.

${}_6C_2$ represents choosing 2 people from the remaining 6 and ${}_4C_4$ represents choosing the last 4 people to clean the gym.

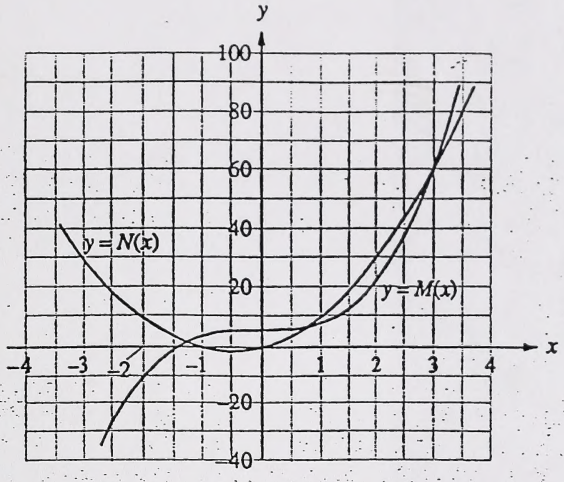
Model B

8! represents the number of ways the jobs can be assigned to the people.

Since the jobs are being assigned to the people, the jobs are the permutation.
The tasks are not all different therefore it is a permutation with repetitions.

Let the letter C represent checking coats. Let the letter P represent selling pop.
Let the letter G represent cleaning the gym.
We must arrange C, C, P, P, G, G, G, G.
Since there are repetitions we must divide by $2! \times 2! \times 4!$

(5 marks) **4.** All the points of intersection of the graphs of $y = M(x) = 2x^3 + 6$ and $y = N(x) = 5x^2 + 5x$ are shown below.



Find the exact values of x for which $M(x) = N(x)$.

Sample Answer Question 4

$M(x) = N(x)$
$2x^3 + 6 = 5x^2 + 5x$
$2x^3 - 5x^2 - 5x + 6 = 0$

From the graph, $x = 3$ appears to be a root of the equation.

-3	2	-5	-5	6
		-6	-3	6
	2	-1	-2	0

$(x - 3)(2x^2 + x - 2) = 0$

Using the quadratic formula for $2x^2 + x - 2 = 0$ we obtain

$$x = \frac{-1 \pm \sqrt{1 + 16}}{4}$$

$$x = \frac{-1 \pm \sqrt{17}}{4}$$

The exact values of x for which $M(x) = N(x)$ are $x = \frac{-1 \pm \sqrt{17}}{4}$ and $x = 3$.